Wind Power Bidding Based on Chance-constrained Optimization

Jianhui Wang
Argonne National Laboratory

In collaboration with

Qianfan Wang, Yongpei Guan,
University of Florida

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Introduction

- Motivation
  - Increasing penetration of wind power in power systems
  - Impact of wind power on traditional electricity markets
  - Uncertainty in wind power output
  - Wind power bidding strategies
- Handle Uncertainty in Wind Power
  - Pumped-storage (hydro) unit
  - Financial hedging strategy
  - Quick-start thermal unit scheduling

Introduction

- Chance Constrained Optimization
  - Stochastic Programming Framework.
  - Application in power systems
    - load uncertainty
    - transmission planning
  - A chance constraint allows for a definition of a certain probability at which a given portion of the wind power can be utilized.

Bidding Strategies in Electricity Markets

- Market Framework
  - IPPs (Independent Power Producers) and customers submit the bids on day d
  - The market operator provides the market clearing prices of electricity based on supply and demand
  - IPPs need to self-schedule their generation portfolio and decide how much they bid into the market considering possible realizations of uncertain factors such as prices and wind power output

Assumptions

- IPPs are assumed to be price takers in the market.
- IPPs are considered to possess and schedule a number of thermal generators, wind farms and pumped-storage units in energy production.
- The wind power utilization falls into an interval with certain probability.

Model Overview

- Two-stage stochastic programming
- First-stage decisions
  - unit-commitment
  - bidding quantity for the day-ahead market
- Second-stage decisions
  - optimal dispatch (thermal, hydro and wind)
  - imbalance penalty
- The chance constraint is considered at the second stage.

Mathematical Formulation

- Objective Function

  $max \ -\sum_{t=1}^{T}\sum_{i\in TG}(SU_i o_{it} + SD_i v_{it})$ +E[Q(o, v, y, ξ)]

- First Stage
  - Min-on/off
  - Start-up/Shut-down
- Second Stage

  $$Q(o, v, y, \xi)=max \sum_{t=1}^{T}(R_t(\xi)\left(q_t^B + q_t^{imb}(\xi)\right) - F_c(x_{it}(\xi)) - \gamma_t|q_t^{imb}(\xi)|)$$

  - Generation upper/lower bound
  - Ramp up/down
  - Generation balance
  - Hydro
  - Wind (Chance Constraint)

Chance Constraint Description

- The chance constraint considers the joint probability which is at least $1-\epsilon$ chance the usage of wind power is larger than or equal to percentage for every operating hour.

- Formulation
  - $Pr(\cap_{t=1}^{T}\{\beta W_t(\xi) \leq q_t^W(\xi)\}) \geq 1-\epsilon;$
  - β is the utilization of wind power;
  - ϵ is the risk level.

Sample Average Approximation (SAA)

- Basic Idea
  - the true distribution of wind power generation is replaced by an empirical distribution using computer simulation.
- Structure
  - Scenario Generation -> SAA;
  - Convergence Analysis;
  - Solving SAA Problem -> Approximated Optimal Solution;
  - Solution Validation -> Solution Quality.

Scenario Generation

- Monte Carlo Simulation
  - We use Monte Carlo simulation to generate scenarios;
  - Assume the wind power is subject to a multivariate normal distribution for every time period t.
- SAA Problem
  - After the scenarios are generated (e.g. N scenarios), we can get the approximated problem; (i.e. SAA problem)
  - *E[Q(o, v, y, ξ)]* is estimated by $N^{-1}\sum_{j=1}^{N} Q(o, v, y, \xi^j)$ [1];
  - The chance constraint can be estimated by an indicator function $N^{-1}\sum_{j=1}^{N} 1_{(0,\infty)} G(x(\xi^j), \xi^j)) \leq 1 - \epsilon$ [2].

Scenario Generation

- Proposition: As the sample size N goes to infinity, the objective of the SAA problem converges to that of the true problem.
  - Proof sketch:
    - Convergence of the chance constraint
    - Convergence of two-stage stochastic programming

Solution Validation

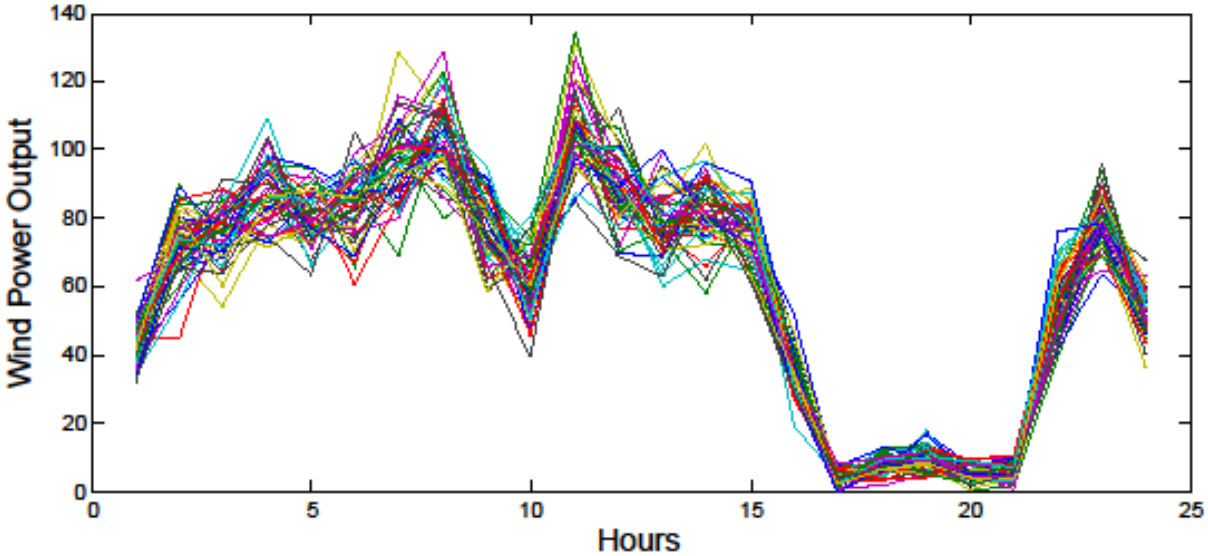
- Basic Idea
  - Assume that x is an optimal solution for the SAA problem; and v is the corresponding objective value.
  - Solution validation provides a scheme to validate its quality by obtaining upper and lower bounds for the corresponding solution.
- Method to obtain Statistical Upper/Lower Bound
  - We introduce a group of Notations
    - N is the scenario size of the SAA problem, M is the iteration number, N' is the validation process to obtain a lower bound;
    - $\hat{g}$ is the lower bound of the true problem; is
    - $\bar{x}_m$ is the optimal solution
    - $\bar{v}_m$ is the optimal objective value in iteration m ;
    - $\bar{v}$ is the upper bound of the true problem.

Solution Validation

- Set m=1, 2, …, M and repeat the following steps for each m:
  - For a given sample size N, generate a corresponding SAA problem and solve the SAA problem to obtain $\bar{x}_m$ and $\bar{v}_m$
  - For a given sample size N' for the validation process, generate independent scenarios and estimate the lower bound of the problem using the following formula:
  - $\hat{g}^m = f(\bar{x}_m) + 1/N' \sum_{n=1}^{N'} Q(\bar{x}_m, \xi^n)$
- Take the average of $\bar{v}_1, \dots, \bar{v}_m$ as the upper bound.
- Take the maximum of $\hat{g}^1, \hat{g}^2, \dots, \hat{g}^m$. The lower bound can be obtained as $\hat{g} = \max_{1 \le m \le M} \hat{g}^m$
- Obtain the optimality gap.

Case Study

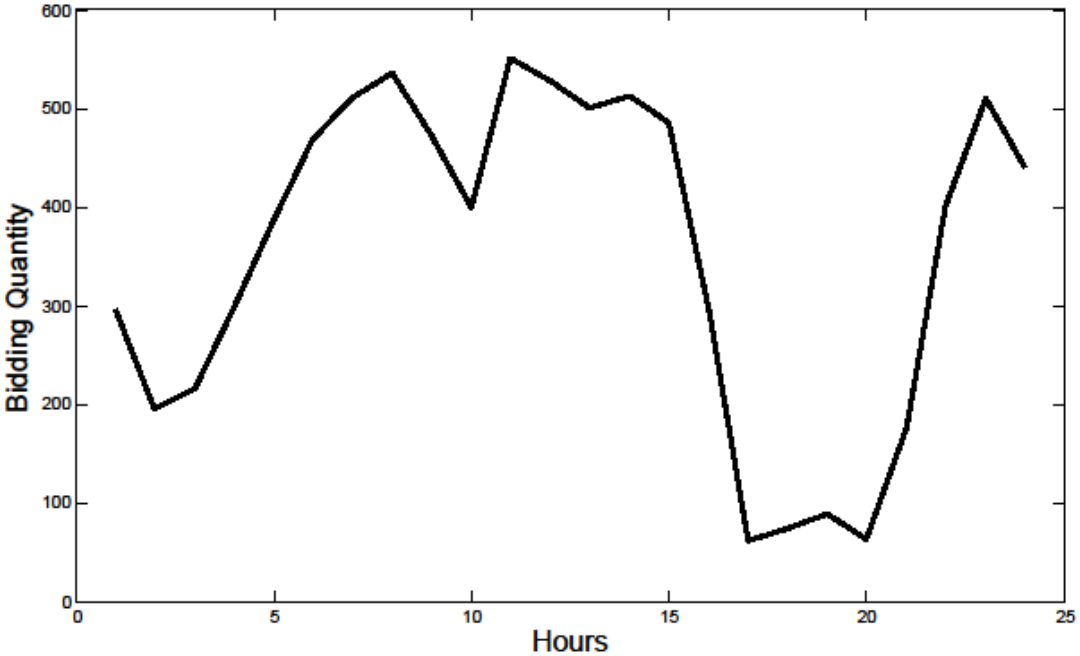
- Simple System
  - Three generators, one wind farm, one hydro unit.
  - Wind Power: multivariate normal distribution



Case Study

- Simple System
  - Optimal solution with ten scenarios
    - Optimal unit commitment (1-24 hour)
    - Bidding curve

G1	1 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
G2	1 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0 1 1 1 1
G3	1 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 0 0 0 0 0 1 1 1



Case Study

- Simple System
  - Optimal solution with ten scenarios
  - It can be observed that G1 is committed most of the time
    - G1's fuel cost is low,
    - more flexible lower/upper bounds and ramp limits
  - It can be observed that the differences in bidding decisions in each time period are relevant to the volatility of the wind power and prices.
    - Example: the bidding quantities are the lowest during hour 16 to hour 20, when the wind power is much lower than the other periods.

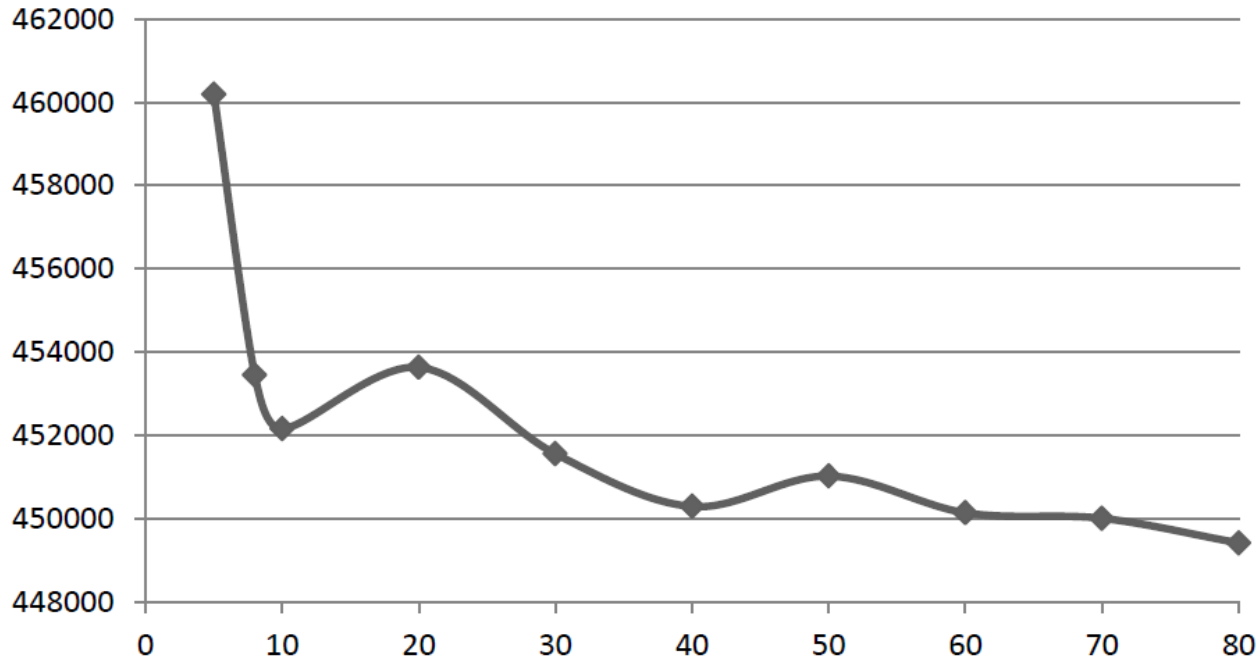
Case Study

- Experimental Results
  - Computational Results for the 3-generator System with different risk levels.
  - The total profit will be lower if the utilization requirement of wind power output is more restrictive.

Risk Level ϵ	Obj.($)	CPU Time(s)
0%	449000	2.88
10%	451551	64.64
40%	456497	501.85
70%	459574	207.00
100%	461785	2.49

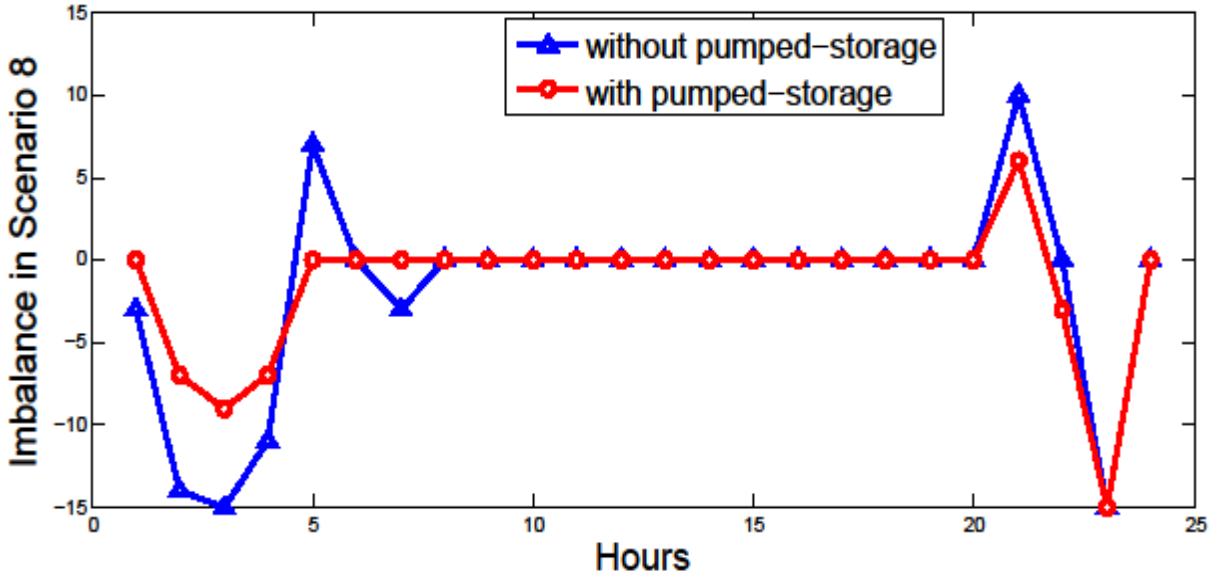
Case Study

- Experimental Results
  - Experiments at different scenario sizes
  - the optimal objective value for the SAA problem converges as the sample size increases.



Case Study

- Experimental Results
  - To show the effectiveness of the hydro unit, we compare the imbalance value in one scenario (scenario size is 50 in this experiment) with and without the hydro unit.



Case Study

- Experimental Results
  - Statistical upper/lower bound by solution validation
  - It can be observed that the gap decreases as the values of M (number of iterations) and N' (number of scenarios) increase.

(M, N')	LB	UB	Gap	CPU Time(s)
(5, 50)	452297	454010	0.37%	210.6
(20, 100)	452540	453908	0.30%	830.5
(50, 200)	452913	453780	0.19%	2005.2

Computation Issues

- Complicated System
  - Scenario Size.
  - Number of thermal unit, wind farm and hydro unit.
  - Computation complexity increases significantly.

- A Heuristic Method
  - Upper Bound
    - Get an upper bound for the max. problem by relaxation.
    - Integer variables lead to the difficulty of solving our problem.
    - Relax the integrality for hydro constraints.

Computation Issues

- A Heuristic Method
  - Lower Bound
    - Get an lower bound by finding the feasible solution;
    - First-stage solution from the above part should satisfy the first-stage constraints in the SAA problem;
    - Fix the first-stage solution obtained from the above part, and solve the second stage sub-problem to obtain a feasible solution.

Computation Issues

- Computational Result
  - It can be observed from Table I that CPLEX cannot solve the original model to optimum within the predefined time limit (2 hours), when the scenario size is larger than 150.
  - However, as Table II indicates, the upper and lower bounds can be obtained by our proposed method within acceptable CPU time.

Table I

N	Obj.	CPU Time(s)	Var. Num.	Con. Num.
50	1482420	11.55	16034	35289
100	1481150	458.58	31684	68889
150	1480070	5025.05	47334	102489
200	-	time limit exceeded	62984	136089

Table II

N	LB	UB	CPU Time(s)
50	1482420	1482420	6.56
100	1481150	1481150	98.25
150	1480070	1480070	391.13
200	1478570	1478570	1961.08

Computation Issues

- Computational Result
  - Heuristic-based Solution Validation
    - Statistical upper bound can be obtained based on the upper bound derived from heuristics.
    - Similarly with the statistical lower bound.

(M, N')	LB	UB	Gap	CPU Time(s)
(10, 200)	1466820	1482320	1.05%	66.5
(30, 500)	1470050	1481070	0.75%	309.8
(50, 800)	1470760	1478650	0.53%	715.3

Conclusion

- A stochastic chance-constrained optimization for wind power bidding strategies is presented
  - Price taker.
  - Uncertain wind power.
- Chance constraint is applied to the wind power utilization constraint.
- Solution Method: SAA

Q&A

- Thank you!
- Jianhui.wang@anl.gov